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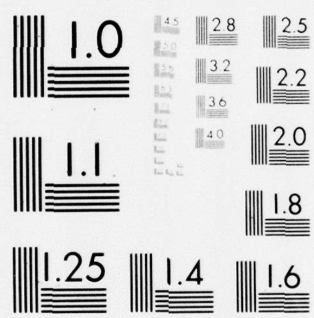

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DETERMINATION OF LOSSES

IN T-BURNER WITH CIRCUMFERENTIAL

SLOTTED VENT

Prepared for

Air Force Office of Scientific Research/NA
Bolling Air Force Base, D.C. 20332

by

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This research program was concerned with an attempt to resolve the controversy surrounding the description of the vent of the T-burner. That controversy, whether there arises a gain or a loss of acoustic energy through the vent, is fundamental to an understanding of the evaluation of T-burner results. The research program was to have involved developing an improved analysis along with a small experimental program to verify that analysis. The program has been terminated.

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ABSTRACT

This research program was concerned with an attempt to resolve the controversy surrounding the description of the vent of the T-burner. That controversy, whether there arises a gain or a loss of acoustic energy through the vent, is fundamental to an understanding of the evaluation of T-burner results. The research program was to have involved developing an improved analysis along with a small experimental program to verify that analysis. The program has been terminated.

TABLE OF CONTENTS

1. INTRODUCTION

Combustion instability is a direct result of the interactions-invariably nonlinear-between the flow field, combustion and wave motions in rocket motors. The T-burner (Ref. 1) is recognized as a generally reliable and convenient tool for the study of the combustion instability phenomena (Ref. 2). In spite of several limitations, the double-ended, center-vented T-burner has provided useful data and shows promise for improvement as its performance becomes better understood.

At this date, however, there are still basic questions pertaining to the burner geometry, the propellant grain geometry, the instrumentation and the data reduction system (Ref. 3). Such questions may be resolved only by a detailed analysis of the performance of a T-burner.

It has been repeatedly pointed out that one of the more important questions in the interpretation of the indicated gain or loss in a T-burner and "...by far the most difficult task is to formulate a realistic representation of the behavior of the subsonic lateral vent generally used in the motor..." (Ref. 4).

The controversy about the effect of the vent, whether there arises a gain or a loss of acoustic energy through it, is fundamental to an understanding of the evaluation of the T-burner results (Refs. 5 and 6). The controversy may be very well observed in the reported results of analysis and certain definitive, albeit simple, experiments (Refs. 7, 8 and 9).

At the same time it should be pointed out that the one-dimensional flow approach to the problem of the vent is entirely open to question; the flow field is three-dimensional in nature and viscous effects cannot be ignored to any order in the corner region.

The objective of this research program was to attempt a resolution of the vent controversy through an improved analysis. In addition, a small experimental program was to be undertaken to verify that new vent model.

II. BACKGROUND

We shall set the background for further investigation by answering briefly the questions: (1) what is the nature of the approaches taken to study instability in T-burners, (2) what is the nature of the assumptions introduced in dealing with the vent, and (3) what are the experimental approaches taken to resolve the vent controversy.

T-burner Instability

The general approaches taken in the study of T-burner instability may be said to be the following: (1) One-dimensional flow with mass addition; (2) One-dimensional flow with varying area of burner cross-section and with mass addition; (3) Three-dimensional flow in the "classical" sense; (4) "Patched" three-dimensional flow to account for coupling between wave motions and boundary processes; and (5) Nonlinear analysis including a nonlinear transient burning rate (Ref. 10). The analyses under (1) and (2) do not require any special discussion (Refs. 11 and 12).

The three-dimensional flow analysis that has been discussed in Ref. 13 is an inviscid analysis. The one-dimensional flow analysis is also an inviscid analysis but contains the mass addition terms explicitly. In order to account for the interactions at the boundaries in the three-dimensional flow analysis, either one has to include all of the boundary processes or one has to set up an analogy between the one and

the three-dimensional flows and identify the interaction terms. What has been achieved in Ref. 13 is the establishment of the boundary conditions that should be explicitly identified in the three-dimensional flow analysis (for example, in the wave number equation) on the basis of a direct analogy with the corresponding one-dimensional equation. That is certainly a useful approach (a) in clarifying what the inviscid three-dimensional flow analysis without explicit mass addition terms can yield and (b) in interpreting the implications of the mass addition terms in the one-dimensional flow analysis. At the same time, it should be clear that no three-dimensional flow analysis including the boundary processes is available to date.

In Ref. 5, it has been stated that, for example, the acoustic/mean flow interactions associated with the flow entering along the lateral boundary "...can be handled in an extension of classical three-dimensional analysis only by including viscous forces and treating essentially a problem of boundary layer theory...".

In the one-dimensional analysis, the input from the lateral boundary is represented by sources of mass, momentum and energy, and the analogy with three-dimensional flows therefore merely identifies those terms.

The growth constant for acoustic waves in a chamber with combustion at the surface and with mean flow has also been obtained (Ref. 14) using the overall energy fluctuation in the chamber. Obviously such an analysis does not account

for the distribution of sources of mass and energy. This has been discussed in Refs. 5 and 13 in some detail. However, the analogical studies with one-dimensional flow have not clarified the situation in any substantial manner.

Also, there is the analysis based on the acoustic energy of the chamber (Ref. 15) wherein the mass generated at the surface is identified as a Dirac delta-function in a thin volume element and is associated with a velocity pertaining to the surface layer region. One can obtain correspondence between a surface process and a volume process, if necessary. That will help clarify the meaning of the 'flow-turning' effect, for example. However, in this lumped-parameter approach, one is essentially carrying out a one-dimensional analysis and not a true three-dimensional flow analysis.

The nonlinear influences in a T-burner arise from nonlinear gas dynamics and velocity coupling. Acoustic pressure oscillations in T-burners reach a limiting amplitude indicating a nonlinear mechanism. A detailed discussion of the current state of one-dimensional nonlinear analysis is available in Ref. 5. Both a numerical solution (method of characteristics) as well as an approximate solution have been obtained. The implications of including nonlinearities, however, are not clear.

Effect of the Vent

It has been stated (Ref. 5) that "...until the importance of the mean flow-associated vent-effect is established, the test technique using pulses during and after burning involves

a potentially serious uncertainty...". As stated again in Ref. 5, "...the three-dimensional unsteady viscous flows at the center exhaust vent and in the vicinity of lips or recessed edges remain unsolved, both analytically and experimentally...".

The state-of-the-art may be discussed in the following manner: (1) Culick's basic analysis, (2) Coates and Horton analysis, and (3) Culick's improved analysis.

In Culick's basic analysis (Ref. 13), the assumption is made that "...there is an inelastic process at the boundary in which the fluid originally participating in the acoustical motions flows out the vent with no acoustic energy...". The immediate consequence is that there arises a gain. However, Culick points out carefully that this is a consequence of the one-dimensional analysis, that it "...in no way constitutes a detailed analysis of the vent..." and also that "...the results are suspect...". At the propellant surface, there is an influx of mass and one correctly obtains the result that a loss occurs under the assumption that the burning mass acquires acoustic speed irreversibly. Yet at the vent, there appears to arise a gain, at least formally, for all modes in a simple uniform burner according to Culick's theory. Of course, one can show, as indeed Culick does, that the effect of the vent vanishes for all modes if one assumes that the axial momentum of the gas leaving through the vent is destroyed by the action of viscous stresses and axial force exerted by the wall of the vent. The contribution to the growth constant due to the resulting force acting on the flow nullifies the gain and the

vent has no net effect.

The Coates and Horton analysis (Refs. 16-18) along with the correlation of experimental results question the aforementioned result and indicate a loss in the nozzle. This is of course based on the assumption that "...the gases leaving the chamber convect the local acoustic energy with them...". Incidentally, Coates and Horton use basically the MHC theory (Ref. 14). They modify the theory to account for the mean flow interaction by invoking the quasi-steady processes approximation (Ref. 19) for the nozzle (briefly, with no distinction between the acoustic and steady components of the velocity of gas passing through the nozzle).

Culick (Refs. 5 and 13) has also considered the radiation of acoustic energy through the vent. Radiation however influences principally only the even modes.

Now, in the analogical studies between one and three-dimensional flows, Culick (Refs. 13 and 20) does take into account the influence of the possible acoustic modes at a rigid surface in terms of the angle between the direction of propagation of the incident waves and the normal to the surface. However, this is not included in the determination of losses at the vent.

In general, the three-dimensional viscous effects and forces exerted by the wall have never been examined in detail at the vent. There is of course a recognition of the nature of the problems involved (Ref. 10): (1) the edge of the vent

acting alternately as a leading and a trailing edge when the flow oscillates and (2) the separation and reattachment of the flow at the vent.

The attenuation of sound waves in a viscous layer can be calculated in a simple fashion either by assuming a locally one-dimensional acoustic boundary layer or by introducing mass sources at the boundary. However, such approaches to the problem preclude the precise nature of the interaction between flow and acoustic motions being correctly assessed. Moreover, such lumped parameter approaches do not provide explicit information for calculations. It should be noted that the diffraction of acoustic oscillations at the vent corner has never been taken into account.

Experimental Studies

Two independent experimental studies, one at the California Institute of Technology and the other at the Naval Weapons Center, previously attempted to determine the acoustic wave/mean flow field interaction in a T-burner. Culick and Magiawala (Ref. 21) at Cal Tech conducted an experimental program concerning the study of interactions between acoustic waves and non-uniform flow fields in a simulated T-burner. Data was taken in an impedance tube and in a resonance tube having the configuration of a T-burner. Their results were inconclusive as to whether a loss or gain can be associated with the exhaust flow through the T-burner vent. The greatest difficulty that they encountered was in measuring precisely

the various losses present in their experimental systems.

Derr and Mathes (Ref. 22) in the second phase of a research program at NWC attempted to conduct cold flow tests in a simulated T-burner to determine the nature of interactions between acoustic waves and mean flow. However, their findings were also inconclusive due to the trouble they experienced in obtaining precise values for the impedances of the end surfaces of their apparatus.

III. OBJECTIVES OF THE PROGRAM

The T-burner to be examined is one with a center-vent extending over the entire circumference of the burner chamber. The propellant grain may be attached to the flat end wall and also to the cylindrical wall. Attention is restricted entirely to longitudinal instability in the T-burner. In addition, the propellant is assumed to be without any metal additives, rather similar to a smokeless propellant.

The acoustic-mean flow interaction is central to the investigation. It is therefore essential to take into account: (a) the state of the gas and the viscosity and heat conduction of the gas; (b) the development of the mean flow in the T-burner, for example on the assumption of axisymmetric, viscous flow conditions; and (c) the compressibility effects introduced on account of Mach number effects. While (a) and (b) are entirely unavoidable in this investigation, the compressibility effects due to flow Mach number and Mach number gradients may be examined under two successive approximations as follows:

(c-1) Negligible Mach number: Terms including Mach number are treated as of lower order.

(c-2) Finite Mach number: The full implication of Mach number-introduced nonlinearities are taken into account. Those nonlinearities interact with other nonlinearities due to instability.

The basic objectives of the investigation are as follows:

(1) To establish in sufficient detail the acoustic-mean flow interaction in the vicinity of the vent; (2) To provide a calculation procedure for the damping coefficient due to the vent; (3) To verify experimentally the acoustic-fluid mechanical interaction in a T-burner configuration without combustion; and (4) To correlate experimental results under (3) above with experimental results obtained from other laboratories on controlled T-burner combustion instability data on the basis of the model and calculation procedure established under (1) and (2) above.

Analytical Studies

The following assumptions are made: (1) Axisymmetric, viscous flow in the burner; (2) Viscosity effects included in the corner turning processes; (3) Acoustic interactions examined in relation to inviscid but vortical motions in the vent corner; and (4) Subsonic flow, with Mach number introduced nonlinearities.

In addition, in order to take into account the mass and heat additions in the motor, it is assumed that the additions arise from distributed sources at the boundaries. The simplest propellant configuration is the one with the propellant confined to the ends. The energy balance is specified in terms of two equations, one for an acoustic energy balance and a second for a global energy balance.

The burner geometry is assumed to be axisymmetric with a vent extending over the entire circumference. The width

of the vent is taken to be equal to about a quarter of the diameter of the tube. The vent corner geometry is characterized by the radius of curvature, one limiting case being a sharp turn.

The only instability that will be examined here is the longitudinal instability in the motor. In some parts of the investigation, a weak shock wave is considered propagating normal to the direction of mean flow and in most of the investigation, only acoustic waves are examined.

It is also assumed that the fundamental mode of oscillation with a pressure node at the vent is the one of interest. With even modes, a pressure anti-node occurs at the vent with no flow and the pressure field is symmetric.

The first problem is to establish the nonsteady viscous flow field in the T-burner using (a) the stream function, vorticity and axial momentum as variables and also (b) the velocities and pressure as variables. It is important to deal with both sets of equations simultaneously.

The second problem is to establish a method of calculating the propagation of a weak shock wave (M = 1.001) in the burner and out through the vent (Refs. 23 and 24). This is a significant part of the investigation. Apart from establishing the changes across a weak shock wave in the course of propagation, this study will indicate the nature of the boundary conditions to be imposed during the acoustic wave interaction studies.

In obtaining the weak shock wave interaction with flow, the stream function-vorticity formulation will be utilized so that details of the viscous flow need not be taken explicitly into account. At the same time since vorticity is convected by the fluid, once the vorticity distribution is known, the propagation characteristics of a weak shock wave can be established along each streamline.

The final portion of the analysis consists in determining the nonlinear damping coefficient utilizing nonlinear axisymmetric equations and numerical methods for the solution of the problem. In this connection, the following problems will have to be solved: (a) identification of appropriate boundary conditions; (b) determination of the most appropriate numerical technique; and (c) establishing the optimum method of including viscous flow field effects.

In each of those problems, the crucial factor is the acoustic-mean flow interaction in the vicinity of the vent and in the vent. Careful attention will be given to assumptions and suggestions made by earlier investigations regarding the following: (1) Flow in the vent, (2) Pressure distribution over the vent, (3) The detail to which losses in momentum have to be incorporated in the vent region, and (4) The manner in which Mach number gradients are to be taken into account.

Experimental Studies

The experimental studies to have been undertaken under

this program were those concentrating on the acoustic-flow interactions in a T-burner configuration. Combustion studies were not anticipated. It was expected that results from T-burners with slotted vents and with combustion would have become available for use from other sources.

The experimental studies were to have involved a cold-gas T-burner having a slotted vent (see Figures 1 and 2). The cold-gas was to have been injected into the T-burner "combustion chamber" at those locations normally considered locations for the burning surface of the propellant. The cold-gas T-burner was to have been of such configuration(s) that a range of frequencies of oscillations of the fundamental mode would have been driven. Those frequencies correspond to those of interest for solid rocket propellants.

The experimental study was to have concentrated on the measurements of the acoustic-flow interactions in a simulated cold flow T-burner configuration (see Figure 1). Measurements of pressure and velocity fluctuations both in the main stream region and in the flow turning region of the circumferential vent were to be taken with the loss in acoustic energy through the vent subsequently determined from these measurements. The cold-gas T-burner was to have been of such configuration(s) that several vent corner shapes of interest could have been investigated experimentally. Pressure oscillation measurements were to be made with Kistler Piezotron miniature pressure sensors and velocity fluctuations were to be determined

with a DISA Electronics' hot-wire anemometer equipped with a specially designed sub-miniature, 2 velocity component probe.

Figure 2 illustrates the setup for the proposed flow experiment. Air supplied from the existing laboratory facility was to be metered and injected equally into both ends of the simulated T-burner and exhausted from the apparatus through the circumferential slotted vent into a surge tank and finally to the atmosphere. The facility exists to preheat the incoming high pressure air if necessary. Flow metering was to be accomplished with BROOKS Armored rotameters equipped with electrical transmitting output.

IV. RESULTS

Analytical Results

(a) A survey of literature on acoustics of branched ducts was conducted. It was found that (i) no satisfactory formulation existed for three-dimensional, viscous, non-adiabatic (with heat generation) flow with acoustic type disturbances imbedded in the flow that could account for turning or branching of duct flows (Refs. 1-13) and (ii) no satisfactory method of incorporating appropriate boundary conditions existed at the joining stations in the duct (Refs. 1-13). Appendix I provides additional details.

(b) A preliminary study of the flow configuration in a T-burner and the available experimental results indicated that experiments conducted in the usual manner with conventional measurements were unlikely to yield details of the processes occurring in the vent corner. Detailed pressure and velocity measurements are required over the entire vent corner region before any coupling between the vent and the motor flows could be established. Even then the coupling itself will arise through losses in momentum and energy and therefore the acquired data on pressure and velocity need to be processed as vector flow impulse quantities.

(c) After several months of effort on various formulations for the vent-coupling phenomena, it was decided that the most appropriate was one that would permit a separate accounting of sound energy (production, dissipation and convention) in addition to accounting for mass, momentum and total energy. In that

fashion, it was expected that one could obtain a clear delineation of the local changes in sound energy at various locations in the vent corner. It was also expected that, as in problems of turbulence, the coupling between vent flow processes and motor flow processes would become fully established through the sound energy balance equation. This formulation could have been pursued further at the time the contract was terminated. Appendix II provides additional details.

(d) In order to obtain some preliminary estimate of the magnitude of terms involved in the formulation described under (c) above and also to gain a preliminary understanding of the vent corner processes, an analytical formulation was undertaken of the problem of propagation of a weak normal shock ($M \sim 1.001$) in the vent corner region. As an aside, we established the diffraction of such a wave at the vent (of finite width), and we could determine the rear-ward influence of a propagating weak shock wave set at a fixed angle (between 0 and 90 deg.) to the vent cross-section as it moved from the "leading edge of the vent" to the "trailing edge of the vent". The next logical step would have been the calculation of the changes in the shock wave as part of the wave moved out of the vent with the flow. Appendices III and IV provide additional details.

Experimental Results

Although the project was terminated prior to the completion of the experimental apparatus, there are comments that may be made regarding the expected experimental results. From the

pressure and velocity oscillation measurements were to have been conducted in the main flow region and in the vicinity of the vent, the gain/loss of the simulated T-burner system's acoustic energy due to the exhaust flow through the vent was to be determined experimentally. It is believed that the results of such an experiment would resolve the present controversies regarding the acoustic/mean flow interaction due to flow turning through the vent of a T-burner. In addition to the above results, an attempt was to have been made to map out the entire velocity flow field in the vicinity of the vent in order to determine whether the circumferential vent made the resulting flow field in the experimental apparatus axisymmetric.

The experimental apparatus was being designed, and there are no experimental results to report.

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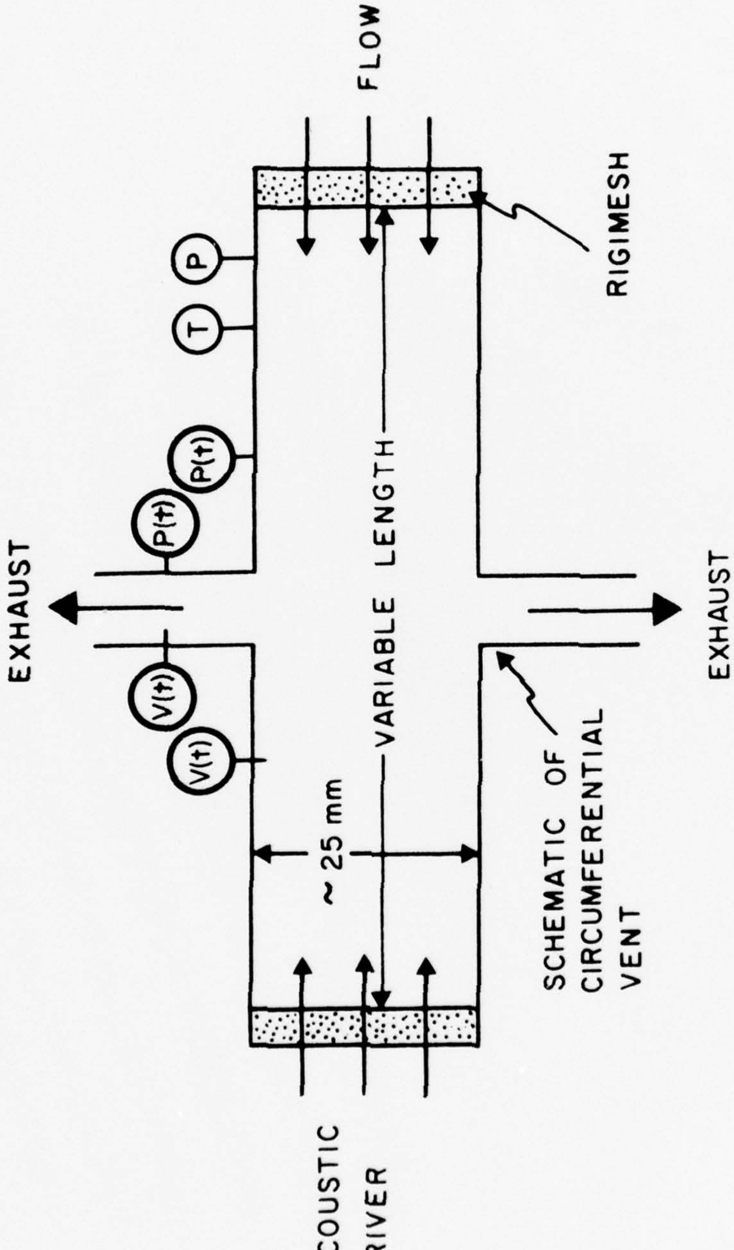
FLOW
RIGIMESH
P
T
P(t)
P(t)
EXHAUST
V(t)
V(t)
VARIABLE LENGTH
~25 mm
EXHAUST TO SURGE TANK
SCHEMATIC OF CIRCUMFERENTIAL VENT
ACOUSTIC DRIVER

FIGURE 1

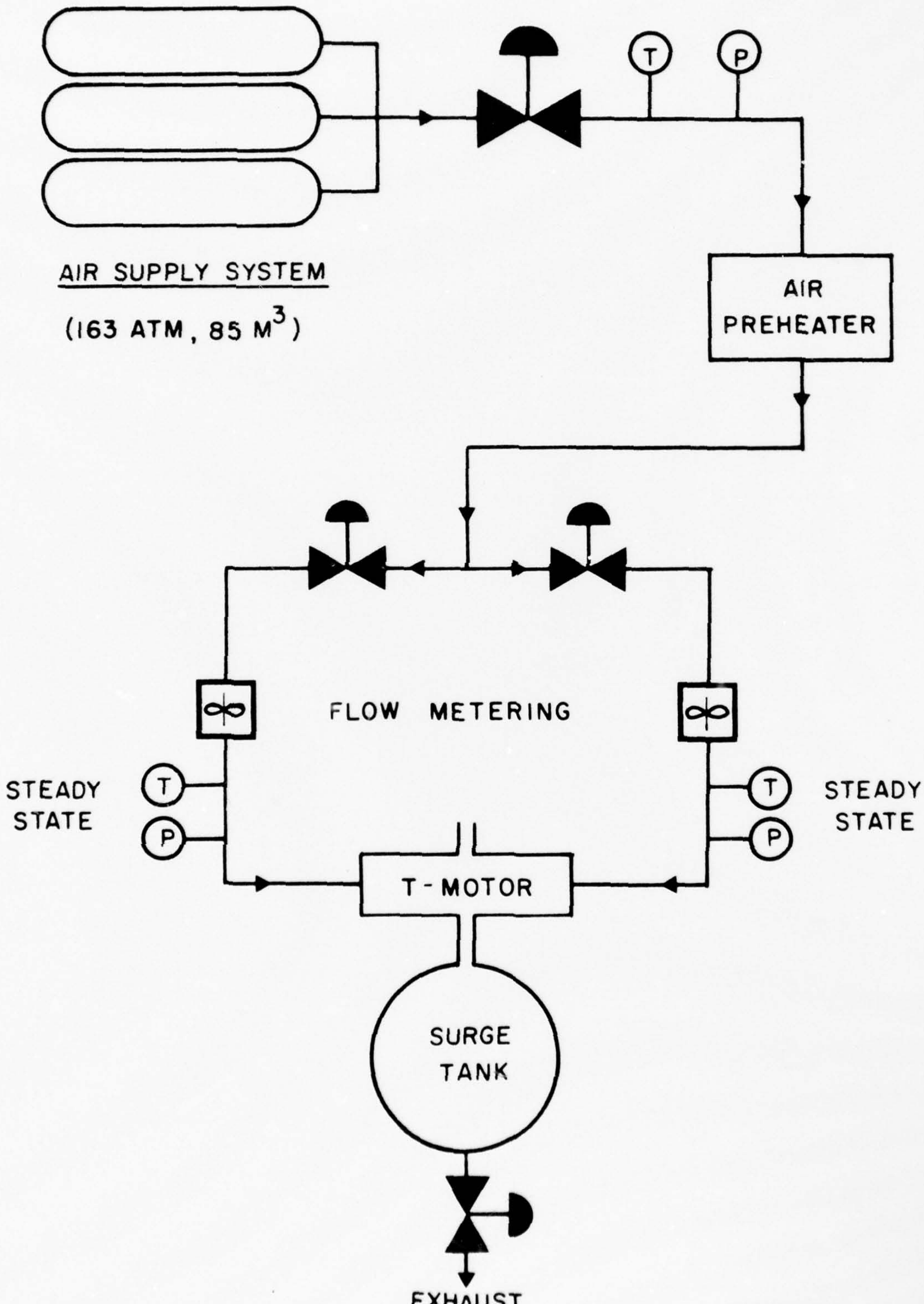
AIR SUPPLY SYSTEM
(163 ATM, 85 M³)
T
P
AIR PREHEATER
FLOW METERING
STEADY STATE
T
P
STEADY STATE
T
P
T-MOTOR
SURGE TANK
EXHAUST

FIGURE 2

Appendix I

Acoustics of Branched Ducts

A literature survey was undertaken to establish the developments in branched duct acoustics that could be applied to the vent problem. While the burner chamber and the vent are both axisymmetric in the T-motor configuration chosen for study, namely a T-motor with a circumferential vent, the joining section between the burner and the circumferential vent may only be analyzed on the basis of fully three-dimensional, viscous, non-constant enthalpy flow. This would be the case whether the vent corner is sharp (separation at the corner) or the vent joining section is faired (separation location becoming part of the solution).

References A.I.1 - I.23 constitute the main literature examined. The following questions were essentially answered in the negative.

(1) Is there a satisfactory way of calculating the steady flow in the vent corner when treated as an elliptic problem?

(2) Does a linear or nonlinear analysis of branched flow acoustics exist that would permit application to the vent corner with simple modification?

(3) Is there an example of the application of proper boundary conditions over a vent of finite width?

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Appendix II

Acoustic Energy Balance

II.1. Background

It is important to recognize in connection with the so-called vent problem that in each of the four or five basic contributions by Crocco, Hart, and McLure, Culick, Horton and Coates, Micheli and Derr (which have been referenced in various places elsewhere in this report), some aspect of the problem has been emphasized more than the others, but each is a perfectly rational and complete solution in the context of the formulation and assumptions adopted.

Three features of the vent flow that are fundamental to a complete solution of the vent problem are (1) the fact that the vent has a finite width and the implications of this for the formation of standing waves and their possible coupling with the vent, (2) the losses occurring in the vent corner and the implications of the distribution of such losses for the assumption of a lumped-loss parameter and (3) the inertia associated with the vent flow and the importance of setting up boundary conditions for velocity and pressure in the vent corner.

It is therefore important to set up three-dimensional, viscous, nonadiabatic flow equations that can be solved with acoustic energy present in the flow and to introduce appropriate boundary conditions to couple the vent and the duct flows.

It can be seen from Ref. A.II.1 that the introduction of sound waves into a duct filled with a viscous fluid has effects on the velocity distribution depending directly upon the ratio

of the reduced frequency, k, and the shear wave number, s, those being defined by

$$k = \omega r / C_o$$

$$s = R\sqrt{P_m \omega / \mu}$$

where R is the radius of the duct and P_m, μ and C_o are the mean density, viscosity and undisturbed acoustic velocity of the gas. The appropriate Kirchoff equation is obtained in that reference in the following form

$$F(r, s, k, \sigma, \gamma) = 0$$

where r is the propagation constant, σ, the square root of the Prandtl number and γ is the ratio of specific heats. This analysis, of course, is restricted to the case with no mean flow.

When the shear number is large, that is when $k/s << 1$, it has been shown in Ref. A.II.1. that the nature of the solutions for velocity distribution is as shown in Fig. A.II.1, where V_x and V_r represent the axial and radial velocities.

This analysis is presented here to show the complexities that will arise when centrifugal forces are present as at a turning of the duct. A solution for such a case with a turning of the duct has never been undertaken.

The foregoing is an analysis of duct acoustics in the absence of motion. If there is mean gas motion, it will become coupled nonlinearly in the Kirchoff equation. If furthermore, the mean flow includes vorticity and vorticity changes, as in a curved flow (vent corner, say), the vorticity fluctuations introduced by acoustic waves will become coupled to the Kirchoff

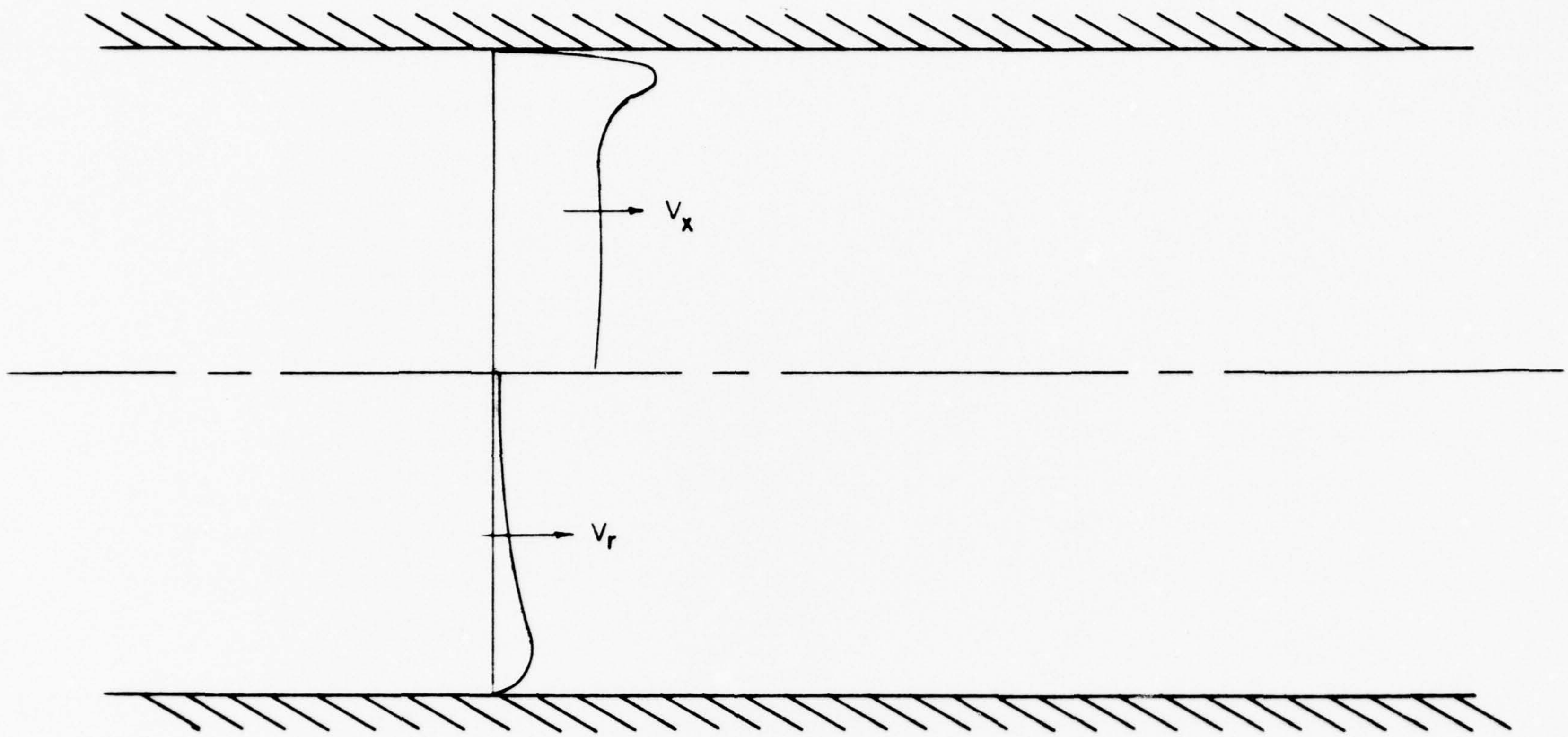

FIGURE A. II. 1.

equation.

The importance of boundary conditions in connecting the vent to the duct in a T-motor may be seen from the following simple example of a branched duct, Fig. A.II.2. The incident (i), reflected (r), branched (b) and transmitted (t) waves are shown in that figure. In order to establish the acoustical motion in such duct, it is necessary to introduce at the vent the following boundary conditions:

a. continuity of pressure; and

b. continuity of volume velocity.

If harmonic acoustics is assumed, e.g.

$P_i = A_i \exp(i\omega t)$, etc.

$u_i = P_i/\rho_o CA$,

where A is the cross-sectional area, the boundary conditions become

i) $P_i + P_r = P_t = P_b$, and

ii) $u_i + u_r = u_t + u_b$

One can then obtain easily the expressions for acoustic impedance and power coefficient of the side branch and the transmitted branch. However, calculation of the impedance and the power coefficient requires the details of the flow process in the three branches of the pipe.

It may be pointed out that the problem of incorporating boundary conditions is not overcome when one assumes "plug" flow in the vent because in order to connect the vent plug to the duct, either one has to assume that there are no corner effects or include the corner effects to sufficient detail.

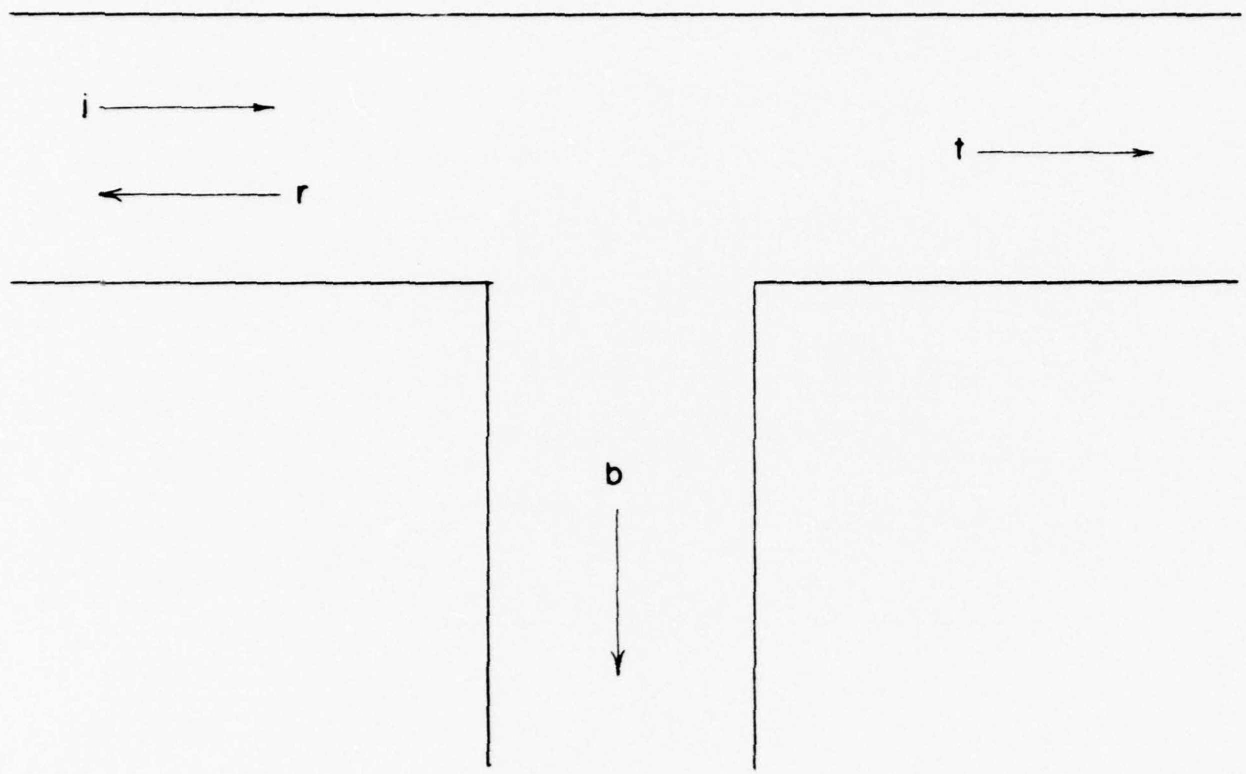

FIGURE A.II.2.

II.2. Acoustic Energy

With that background, it has been concluded that the only method of establishing the "vent effect" is to solve for acoustic impulse or acoustic energy in the entire vent region, comprising of some distance upstream of the vent corner, the vent corner and some distance downstream of the corner. It was decided that acoustic energy is possibly the more meaningful parameter for purposes of calculation (and also measurement).

We shall define acoustic energy presently. Meanwhile, the vent problem is formulated as follows. The flow equations and boundary conditions are written for three dimensional flow, using intrinsic coordinates*, and incorporating viscous effects, heat addition and acoustic energy. Those equations may possibly be reduced to a parabolic form (initial value problem) or they may have to be retained in an elliptic form (boundary value problem). It is also possible that by suitable modifications, the equations may be recast in hyperbolic form but our current thinking was it may prove extremely unwieldy or capable of application only in simple cases (see Appendix IV).

The philosophy of approach in solving such equations with acoustic energy as part of the energy balance equation (with corresponding terms in the momentum balance equation) is as follows: the vent effect arises on account of the coupling between flow

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*Intrinsic coordinates are probably the best choice in view of the fact that, while the T-motor duct is axisymmetric, flow in the vent is radial even when a circumferential vent is considered.

and acoustics in the duct and in the vent. It is therefore important not to separate the acoustic calculations from the flow calculations. One method of doing this is to solve the combined equations.

Now, the acoustic energy may be defined as follows: Consider an acoustic pressure perturbation given by the following.

$$\hat{\underline{p}}\ (x) = |\underline{A}|\ \exp\ [\nu\underline{x} + i\Delta - ik\underline{x}]$$
$$+ |\underline{B}|\ \exp\ [-\nu\underline{x} - i\Delta + ik\underline{x}]$$

with

$$\underline{B} = -\underline{A}\ \exp\ (2\psi)$$

$$\underline{\psi} = \pi\ (\underline{\alpha_o} + i\beta_o)$$

Then, the energy of such a wave system is given by

$$E\ (\underline{x},t) = \frac{1}{2}\ [\frac{\underline{p}^2}{\rho_o c^2} + \underline{U}^2]$$

Suppose now that the acoustic energy is considered as one of the dependent variables. Then an acoustic energy balance equation can be constructed as follows.

$$\frac{D\dot{E}}{Dt} = \text{production of E + dissipation of E}$$

The production terms can then be broken up into

i) mean flow work on acoustic stress and

ii) product of fluctuating pressure and fluctuating strain rate

The dissipation is simply equal to the energy loss in overcoming viscous stresses.

Utilizing the above definitions, one can set up an acoustic energy balance equation which is now treated as one of the set

of conservation equations, the five fluid mechanical equations and the two acoustic equations (one being the definition of acoustic energy and the other the acoustic energy balance equation).

When that system of equations is solved, we should have a map of acoustic energy distribution in the whole of the T-motor (at least in the vicinity of the vent) and one should be able to establish the gain or loss of acoustic energy in the duct for various duct and vent conditions.

It may be stated that the actual equations were not written down in the period of the contract.

Appendix III

Diffraction of a Weak Shock Wave

1. Problem formulation

We consider a weak shock wave propagating in an ideal gas along an unbounded plane with a crosswise rectilinear slit of width 2b in the plane. We wish to establish the diffraction of the wave by the slit. Reference III.1 provides an analysis of a wave moving parallel to the plane and the current analysis consider a wave front inclined to the plane.

The problem is based as an initial-boundary-value problem with a moving boundary for the two-dimensional wave equation.

The following assumptions are introduced:

(a) The flow is two-dimensional and irrotational.

(b) The wave front is represented by a plane moving with a velocity equal to that of sound, c.

In a rectilinear coordinate system (x,y), the wave is assumed to impinge on the slot from the lower half-space. The velocity vector of the wave front is assumed to form an angle θ $(0 < \theta \leq \pi/2)$ with the plane of gas flow. We consider throughout the case where $\theta < \pi/2$.

We define the following quantities:

(a) velocity potential of the wave = ϕ_{ω} (x,y,t)

(b) velocity potential of flow = ϕ. Hence $\phi_y = 0$ at points on the plane.

(c) velocity potential of the perturbed gas flow = Φ where

$$\Phi = \phi_{\omega} + \phi \qquad (1)$$

Hence, the wave equation becomes

$$\Phi_{xx} + \Phi_{yy} - \frac{1}{C^2} \Phi_{tt} = 0 \tag{2}$$

It is clear that ϕ satisfies in the diffraction zone Eq. (2). It is the principal unknown in the problem.

The potential ϕ is the subject to the following conditions:

(1) $\phi\ (x,-y,t) = -\phi\ (x,y,t)$ (3)

(2) on the x-axis,

(a) in the plane behind the incident wave front

$$\begin{aligned}\phi_y &= -[\phi_{\omega y}\ (x,y,t)] \\ &= A\ (x,t)\end{aligned} \tag{4}$$

(b) in the slit

$$\phi = 0 \tag{5}$$

In order to solve the problem, we consider the space of the variables x,y,t. In the (x,t) plane we identify, as in Fig. A.III.1, three domains, Σ_1 and Σ_2 in which the condition (Eq. 4) is specified and Σ_0 in which the condition (Eq. 5) is satisfied.

The domain Σ_1 is bounded by the line W and line L_1. The line W represents the motion of the point intersection of the incident wave front with the x-axis, point A in Fig. A.III.1. The line L_1 is parallel to the t-axis and is separated from it by a distance b.

The domain Σ_2 is bounded by the line W and the line L_2 which is again parallel to the t-axis and is separated from it by a distance b.

It will be observed in Fig. A.III.1. that the domains

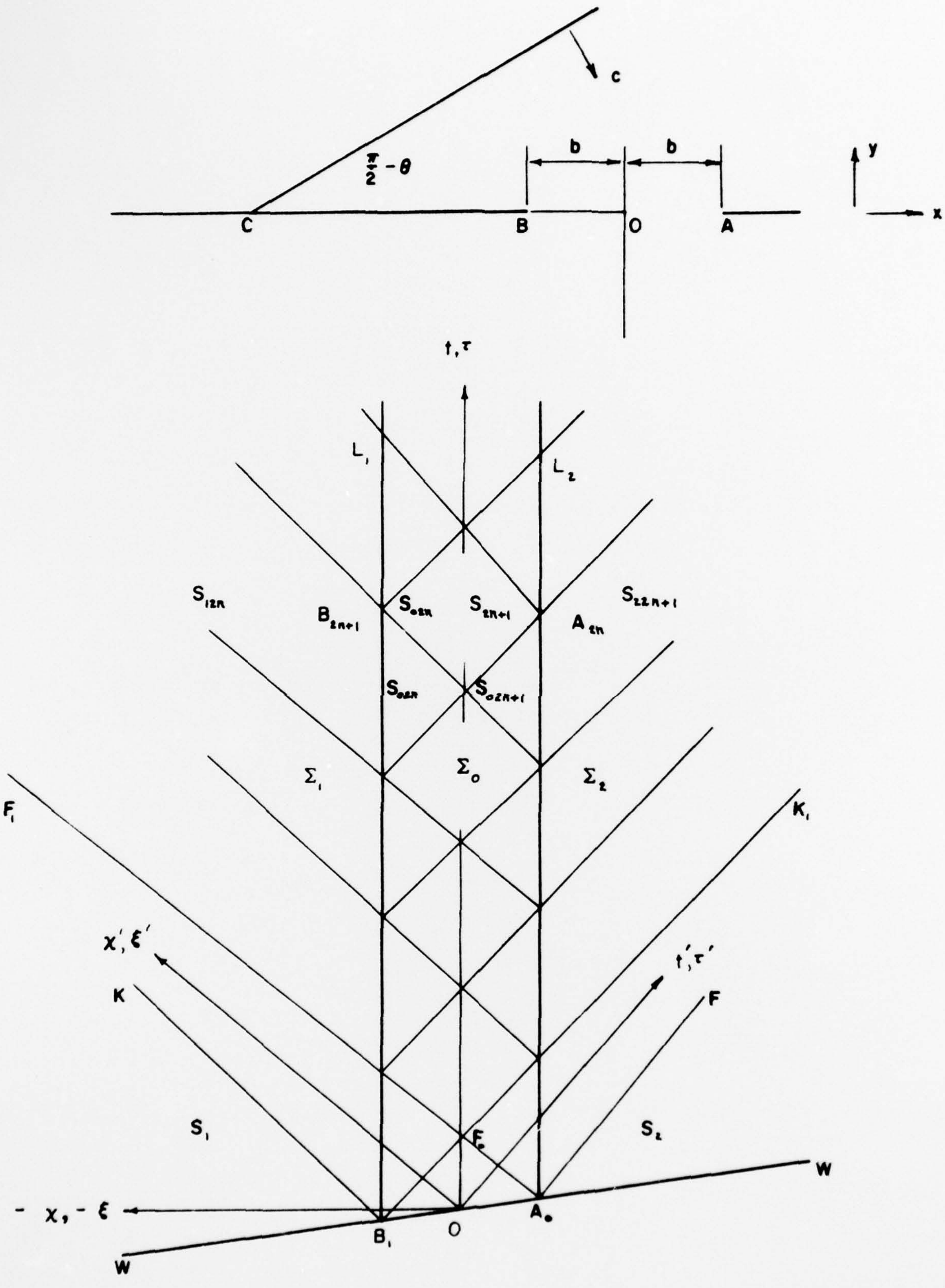

FIGURE A. III. 1.

Σ_0, Σ_1 and Σ_2 are further divided into subdomains S_{o2n}, S_{o2n+1}, $S_{o2n,2n+1}$; S_1, S_{12n}; and S_2, S_{22n+1} respectively. Such a division is necessary because the solutions in those subdomains are analytically different. The points A_{2n} $(+b, t_{2n})$ and B_{2n+1} $(-b, t_{2n+1})$ correspond to A and B respectively at the instants of time when cylindrical diffraction waves are generated there locally, those waves being represented by n = 0,1,2,.... The characteristic cones corresponding to Eq. (1) with vertices located at points A_{2n}, B_{2n+1} divide the (x,y,t) space into the analytically different domains.

The region in which the effect of the slit is felt can be determined as follows. The characteristic cones with vertices situated at points A_o and B_1 bound the region in which the influence of the diffraction waves generated at the slit is felt. Thus, the region where the influence of the slit is felt in the (x,t) plane is the region A_oF, B_1K_1, A_oF_o, F_oB_1 in Fig. A.III.L.

In order to determine the flow potential at any point in the region of influence of the slit, we proceed as follows:

Define a new coordinate system (x', y', t') given by

$$c\,(x' - t') = 2x$$

$$cy' = y$$

$$x' + t' = 2t$$

Let ϕ' and $\phi'_{y'}$ be the new velocity potential and its derivative in the new variables.

We can then write the solution to Eq. (2) in the form

$$\phi'\,(x', y', t') = -\frac{1}{2\pi} \iint_{S_2} \left[\phi'_{y'}(\xi', y', \tau')_{y'=0} \right.$$

$$\left. \frac{d\xi' d\tau'}{\sqrt{(x'-\xi')(t'-\tau')y'^2}} \right] \qquad (6)$$

In order to calculate the velocity potential ϕ' at any point situated in the region of influence of the slit, it is necessary, therefore, to find $\phi'_{y'}$ in domain Σ_o.

The general solution procedure beyond this point can be based on the application of the Abel integral approach as in wing theory.

2. Acoustic pressure calculation

We now proceed to determine the acoustic pressure on the plane in the time interval

$$-\frac{b}{c} \cos \theta < t < \frac{b}{c} (2-\cos\theta) \qquad (7)$$

Utilizing the Lagrange integral for unsteady rotational gas flow, the pressure difference at any point of the plane can be written as follows.

$$\begin{aligned} p\ (x',t') &= p_+ - p_- \\ &= 2P\ [\theta'_{x'}(x',t') + \theta'_{t'}(x',t')] \end{aligned} \qquad (8)$$

where p_+ and p_- are the gas pressures on the upper and lower sides of the plane.

Consider now the subdomains s_o and s_i. The subdomain s_o is characterized by the fact that it is the domain in which the influence of the diffraction wave generated at the boundary A of the slit at time

$$t_o = \frac{b}{c} \cos \theta$$

is felt. The radius of the wave front is given by

$$\nu_o - c\ (t - t_o).$$

There, the pressure difference in the domain s_o is given by the following.

$$p\ (n',t') = -\frac{\rho}{\pi} \int_{n'+T}^{t'} \int_{-n'\tan^2\frac{\theta}{2}}^{n'} [\alpha] d\xi' d\tau'$$

$$-\frac{p}{\pi} [1 + \tan^2\frac{\theta}{2}] \int_{x'+T}^{t'} [\beta]\ dt' \qquad (9)$$

where $$\alpha = \frac{A'_{\xi'}\ (\xi',\tau') = A'_{\tau'}\ (\xi',\tau')}{[(x'-\xi')\ (t'-\tau')]^{\frac{1}{2}}}$$

$$\beta = \frac{A\ \ (-\tau'\ \tan^2\theta/2,\ \tau')}{[(t'-\tau')\ (x'+\tau'\tan^2\theta/2)]^{\frac{1}{2}}}$$

and $$T = \frac{2b}{c}$$

The subdomain s_1 is characterized by the fact that it is the domain in which the influence of the diffraction wave generated at the boundary B of the slit at time t, is felt. The pressure difference in the domain s, is then given by the following.

$$p(x',t') = \frac{\rho}{\pi} \int_{t'+T}^{x'} \int_{-\xi'\cot^2\frac{\theta}{2}}^{t'} [\alpha]\ d\xi'\ d\tau'$$

$$-\ \frac{\rho}{\pi} [1 + \cot^2\frac{\theta}{2}] \int_{t'+T}^{x'} [\gamma] d\xi'$$

where $$\gamma = \frac{A'\ \ (\xi',\ -\xi'\cot^2\frac{\theta}{2})}{[(x'-\xi')(t'+\xi'\cot^2\frac{\theta}{2})]} \qquad (10)$$

Now, consider Fig. A.III.2. It is clear that the pressure difference on the segment AN,

$$-b - ct + b \cos \theta < x < -b$$

in the interval of time

$$t_o < t < t_2$$

is given by Eq. (9).

Similarly, the pressure difference on the segment CN,

$$-ct/\cos \theta < x < -b-ct + b \cos \theta$$

can also be calculated from Eq. (9) using appropriate limits of integration.

Finally, the pressure difference on the segment BM,

$$b < x < -b-ct + b \cos \theta$$

in the interval of time

$$t_, < t < t_2$$

is given by Eq. (10).

In conclusion, it can be seen that the segment CN is not affected by the diffraction waves from the slit and therefore the pressure difference is only created by reflected waves.

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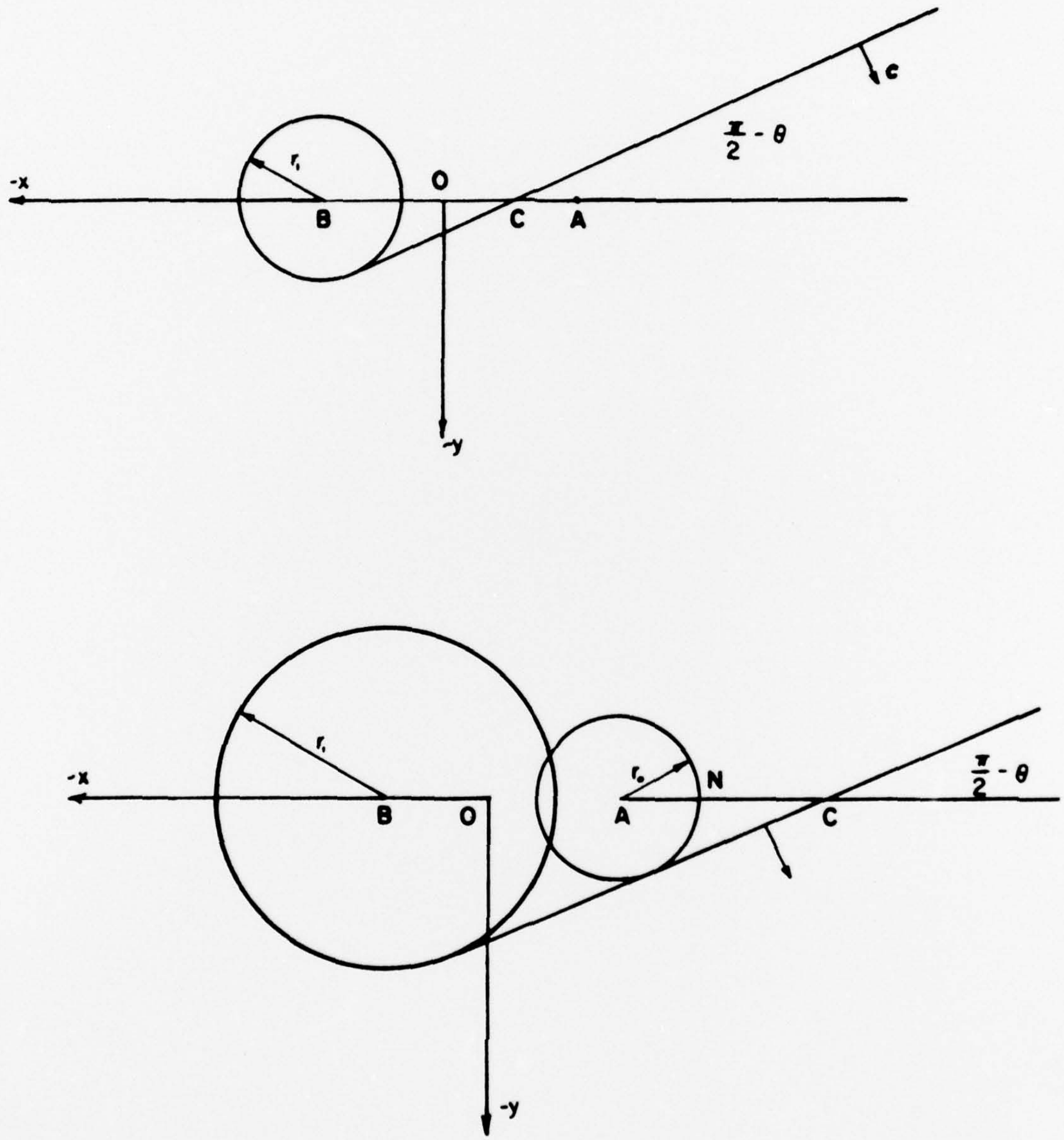

FIGURE A.III.2.

Appendix IV

Shock Wave Propagation in the Vent-Corner

Consider one half of the T-motor as shown in Fig. IV-1 and three stations along the flow at A, C and V.

Problem: To establish the propagation of a weak shock wave from A to V assuming chemically reactive, viscous non-adiabatic, subsonic flow from A to V.

Formulation: The problem is formulated in three parts:

(i) Determination of the flow field from A to V for given free stream conditions.

(ii) Determination of equations yielding changes across a weak shock wave when moving against the flow.

(iii) Setting up of a solution procedure combining the foregoing.

The following simplifications are introduced in a preliminary analysis.

(a) While the viscous, nonadiabatic flow is from calculated in detail, it is replaced with a series of stream tubes in each of which it is assumed that the flow is one-dimensional.

(b) The shock wave is assumed to be locally normal to the stream tubes.

In a more detailed analysis, the interaction between the shock wave and the flow field requires the application of the method of characteristics to a two-dimensional (rotational) flow field with a propagating discontinuity.

Solution procedure: A solution procedure was not determined in the contract period.

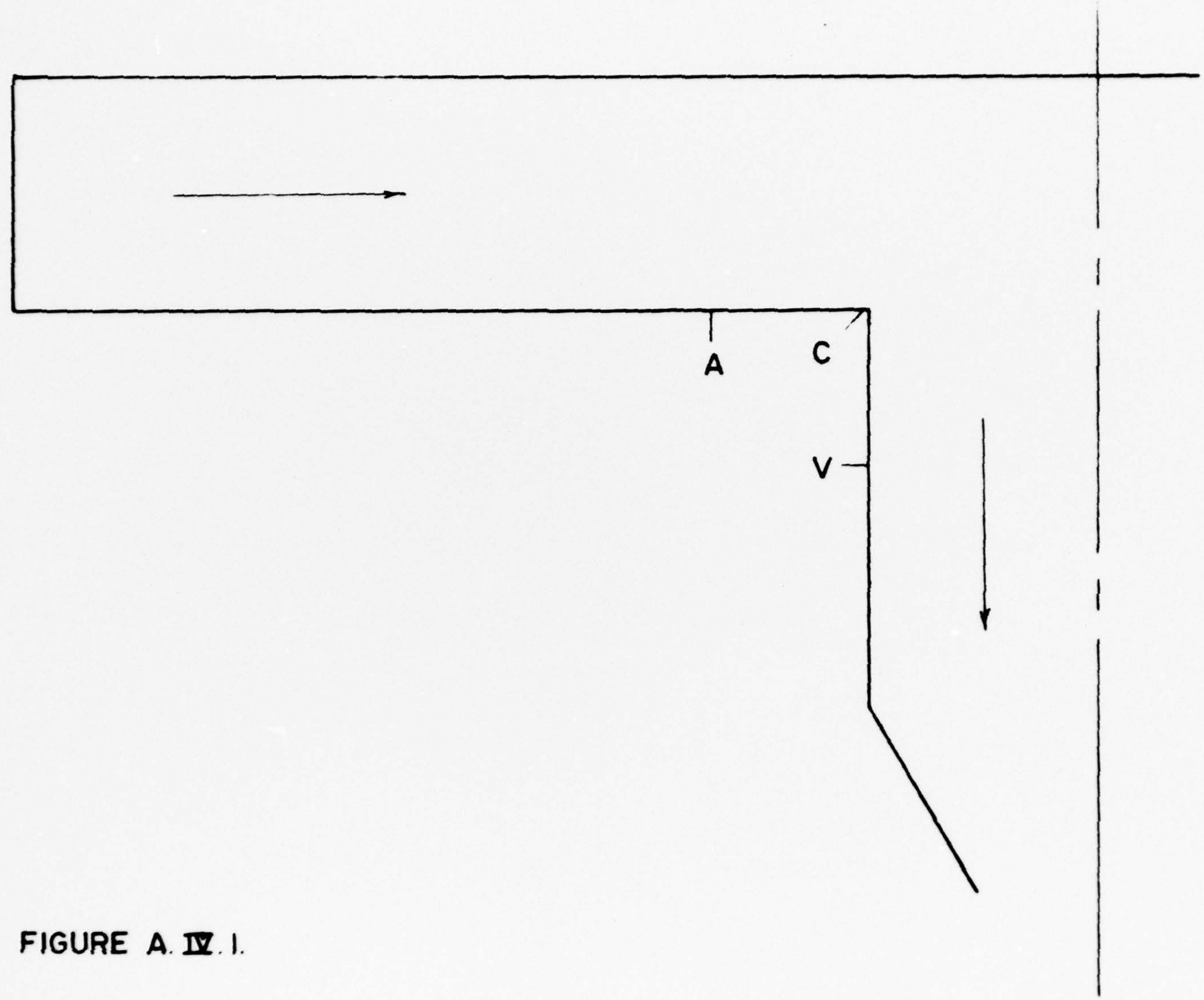

FIGURE A. IV. 1.

However, the following rudimentary analysis should be of interest.

Consider three curved stream tubes as shown in Fig. A.IV.2. W_1, W_2 and W_3 are normal shocklets of very low shock Mach number, each of them locally normal to the stream tubes. Consider the case where the three shocklets are moving with the fluid, as indicated and let this be considered the positive direction for motion of the shocklets. The objective is to establish the changes in the shocklets as they traverse the stream tubes. Calculations can be performed in each stream tube and then the changes in the strength of the shocklets can be integrated on a linear basis to obtain the overall change. As stated earlier this will only provide a first order evaluation of the effect. However, having established this order of magnitude change in the flow and in the strength of the shocklets, one can proceed to the analysis of a two-dimensional rotational flow (as is obtained in a vent corner) and a propagating shock wave.

The changes across a shocklet are calculated using the standard Rankine-Hugoniot equations which may be written as follows for the present problem.

$$p = \rho_s A_s^2\ [2M^2/(\gamma+1) - (\gamma-1)/\gamma(\gamma+1)] \quad (1)$$

$$\rho = (\gamma+1)\ \rho_s\ M^2/[(\gamma-1)\ M^2 + 2] \quad (2)$$

$$U = U_s + 2\ A_s\ (M-{}^1/M)/(\gamma+1) \quad (3)$$

$$A = A_x\ K/(\gamma+1)M,\ K= f(r) \quad (4)$$

$$\begin{aligned} dp = {} & f\ \rho_s A_s^2\ M\ dM/(\gamma+1)\ + \\ & 2\ \rho_s A_s^2\ [2M^2/(\gamma+1) - (\gamma-1)/\gamma(\gamma+1)\ dA_s\ + \\ & A_s^2\ [2M/(\gamma+1) - (\gamma-1)/\gamma(\gamma+1)\ d\rho_s \end{aligned} \quad (5)$$

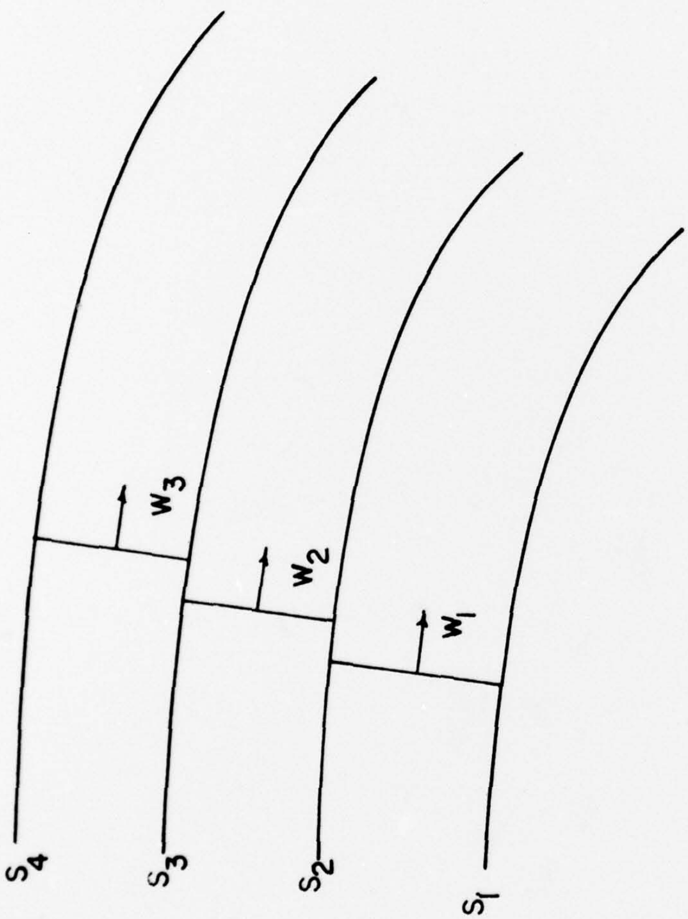

FIGURE A.IV.2.

$$du = dU_s + 2\ A_s\ (1 + {}^1/M^2)\ dM/(\gamma+1) +$$

$$2\ (M-1M)\ d\ A_s/(\gamma+1) \qquad (6)$$

where the subscript s refers to a shocklet and all other terminology is standard.

The analytical solution for determining the effect of gradients in the flow upon the change in the strength of a shocklet is based on Refs. A.IV.1-4, in particular the method proposed by Whitam. It is based on the assumption that changes in unsteady flow behind a propagating normal shock along a positive characteristic can be expected in terms of the steady flow in front of the shocklet and the shocklet strength. This is done by substituting the Rankine-Hugoniot equations and their total derivatives into the compatibility relation of the positive characteristic, namely

$$\frac{dx}{dt} = u + a, \qquad (7)$$

the compatibility relation being

$$\delta{+}P/\delta t + (ae)\ i{+}u/\delta t =$$

$$(\gamma-1)\ \alpha\ \nu_c\ \rho_r \exp(-\varepsilon/t) \qquad (8)$$

where the right hand side incorporates the standard Arrhenius relation for chemical reaction rate.

After some algebra, Eqs. (8), (2), (4), (5) and (6) yield, on removing certain higher order quantities, the following relation connecting the change in Mach number and the changes in temperature, density and heat release due to chemical action.

$$[4M^2 + \frac{2k\ (M^2 + 1)}{(\gamma-1)\ M^2 + 2}]\ \frac{dM}{M}$$

$$= -[2M^2 + \frac{1}{\gamma} - 1 + \frac{k(M^2 - 1)}{(\gamma - 1)\ M^2 + 2} - \frac{H\varepsilon}{2T_s}]\ \frac{dt_s}{T_s} +$$

$$\left[\frac{H}{2} - 2M^2 - \frac{1}{\gamma} + 1\right] \frac{d\rho_s}{\rho_s} +$$

$$H\ \delta x \qquad (9)$$

where

$$H \equiv \frac{\gamma^2 - 1\alpha\ U_c\ Y_r\ \exp\ (-\varepsilon/T)}{MA_s^{\ 2}\ (A_s + d\ A_s/2)} \qquad (10)$$

and Y_r is the reactant concentration, given by (ρ_r/ρ).

Using Eq. (10), it is now possible to proceed systematically, by incremental steps in dr along the stream tube, to calculate the changes in properties of the flow and the changes in the strength of the shocklets.

No numerical work could be undertaken in the period of the contract.

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